Control of quadruple tank system

Introduction

The purpose of this task is to study problems related to multivariable control. The studied quadruple tank process has two inputs and two outputs, and by making minor changes to the process it will switch from minimum phase to non-minimum phase. At the same time the relative gain of the system will change sign, meaning that a feedback controller controlling the process will also need a change in a sign. Or preferably you use different feedback structure for the two cases.

The Process

The quadruple tank process is shown in Figure 1. The goal of the control system is to control the level of the two lower tanks. What makes the task more challenging is that the water from the upper tanks flows down to the tanks below. Pump 1 feeds tank 1 and 4, and pump 2 feeds tank 2 and 3. So we have interaction between the two tanks that are controlled.

![Figure 1. The quadruple tank process.](image)

The inputs $u_1$ and $u_2$ are control signals to pumps, and the outputs $y_1$ and $y_2$ are signals from the sonar's that measure the level in the lower tanks.

The process has the following approximate data:
- Area of the cross section of the tanks: $A_i = 29.225 \text{ cm}^2$
- Area of the holes in the bottom of the tanks: $a_i = 0.1257 \text{ cm}^2$
- Height of the tanks: $h_{\text{max}} = 19.5 \text{ cm}$
- Calibration constant for pumps: $k_i = \text{to be estimated}$
- Calibration constant for level measurements: $k_m \approx 1$ (should be checked)
- Gravitation constant: $g = 981 \text{ cm/s}^2$
Task 1.
Determine a nonlinear model for the tank process based on a mass balance, and linearize the model, and design PID controllers based on the linearized model. You can assume that the in task 7 experimentally determined constants are equal to 1 at this stage (they will only affect the gain of the system, so you can easily compensate for coefficients different from 1 later).

The output (volumetric, as the square root term gives the velocity) flow from each tank follows approximately the law of Bernoulli
\[ q_{o,i} = a_i \sqrt{2gh_i}, \]  
(1)
where \( h_i \) is the water level, and \( a_i \) and \( g \) were specified earlier.

The flows from the two pumps are split up using the valves in Figure 1, so that one fraction of the pumped flow goes to the lower tank and the rest goes to the upper tank. This can be modeled by two parameters \( \gamma_1, \gamma_2 \in (0,1) \). The flow to tank 1 is \( \gamma_1 (k_i u_i) \), and the flow to tank 4 is \( (1 - \gamma_1) k_i u_i \). And similarly the flow to tank 2 is \( \gamma_2 k_i u_i \) and the flow to tank 3 is \( (1 - \gamma_2) k_i u_i \). We will consider two cases, \( \gamma_1 = \gamma_2 = 0.7 \) and \( \gamma_1 = \gamma_2 = 0.3 \). In practice, these two parameters will have to be manually adjusted using the valves in the four flow meters, and it is quite hard to keep the ratios constant.

The measurements from the sonar are assumed to be given by \( y_1 = k_m h_1 \) and \( y_2 = k_m h_2 \), where \( k_m \) is a calibration constant that was checked in task 1, and which should be close to 1.

Now determine the nonlinear model by taking mass balances around each tank. You have water of constant temperature in the tanks, so volume balances will be sufficient.

Task 2. Linearize the above obtained nonlinear model around a operating point \( h_1^0, h_2^0, h_3^0, h_4^0, u_1^0, u_2^0 \), and write it in state space form. Out of these \( h_1^0, h_2^0 \) are controlled and given in the end, \( h_3^0, h_4^0 \) can be calculated by setting derivatives to zero in the differential equations, and \( u_1^0, u_2^0 \) won’t matter (because the variables only appear linearly).

Task 3. Determine transfer functions for the system, and calculate the zeros for these. Test how these are affected by the parameters \( \gamma_1 \) and \( \gamma_2 \), and especially the two considered cases \( \gamma_1 = \gamma_2 = 0.7 \) and \( \gamma_1 = \gamma_2 = 0.3 \). Note that each tank is by them self minimum phase, it is the multivariable system that becomes non-minimum phase, due to the interaction.

Task 4. Build a simulink model of the nonlinear system, and compare it with the linearized model using simulation. Do you get typical non-minimum-phase behavior?

Task 5. Calculate the relative gain array (RGA) for the linear model, and the do variable pairing for the two considered cases (\( \gamma_1 = 0.7 \) and \( \gamma_1 = 0.3 \)).

Task 6. Design PID controllers for the two considered cases, using the IMC method. In one of the cases we have second order systems, when we should not try to get a first order closed loop system, make it a second order system. Tune the IMC closed-loop time constant using simulations.
**Task 7.** Preparation for the lab:

- Check that all pipes are connected and there is enough water in the feed basket.
- Switch on the pumps with the switch in the back of the pumps. Put mode to “ana” (analog) using the buttons in the front of the pump.
- Check that the lower flow meters are open.
- Check that the big valve on the pipe between tank 1 and 2 is **closed**.
- Turn on the computer, and open the simulink model c:\tanklab\tanklaboration.mdl
- Test a constant input, that should be between 0 and 1. An input 0.6 should give a level around 10cm.
- First check the pump flows, both pumps should give the same flow. If not, this is probably due to that the pipes in the peristaltic pumps are worn. Move or replace the pipe if necessary.
- Determine experimentally the constants $k_i$ in the relation $V_i \approx k_i u_i$, where $V_i$ is the observed flow $i$, and $u_i$ is the signal from Matlab to the pumps. Use a sufficient number of different values of $u_i$, and estimate $k_i$ using linear regression. Note that the readings from the rotameters are in \( \ell/min \).
- Also check that the actual levels in the tank are close to the measured levels (that $k_m \approx 1$).

**Task 8.** Test your controllers on the real system.

Start by adjusting the flow meter valves so that 70% of the feed flow goes to the lower tanks and 30% to the upper tanks. All valves must be almost closed so that the effect from height difference becomes smaller. Closing both valves too much will result in the pipe jumping off somewhere.

Then open c:\tanklabb\tanklaboration.mdl, and paste in your controllers. Start the experiment with setpoints \([10 \ 10]\) from start, and change it to \([8 \ 12]\) at 300 seconds, and back to \([10 \ 10]\) at 600 seconds. Remember to save the data, which is stored in the matrices pump1, pump2, LT1 and LT2 (with time in the first column, and the variable itself in the second column).

After this adjust the flows so that 70% goes to the upper tanks and 30% to the lower tanks. Try now controlling the system with the same controller, what happens?

Apply the controller designed for this case, and try it instead.

Make figures of all three experiments, and include them in the report.