Compound Real Options with the Pay-off Method: a Three-Stage R&D Case Illustration

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R&D Case Illustration

Compound real options are combinations of real options, where an exercise of a real option opens another real option. Compound real options are commonly found in a number of industrial projects, but are especially relevant in, e.g., research and development (R&D) where the R&D projects give the real option to research further, or to start the implementation of the results. This paper discusses the valuation of compound real options with the pay-off method for real option valuation and shows that the method is simple to use also in the valuation of compound real options.

Keywords: Pay-off method, real option valuation, compound options, fuzzy logic, R&D

Introduction

Real options (RO) are useful as a mental model for strategic and operational decision-making and as a valuation and numerical analysis tool to support the same. Real option valuation (ROV) is based on the observation that the possibilities financial options give their holder resemble the possibilities found connected to real investments, see, e.g. (Pindyck 1991). Real option valuation is treating investment opportunities and the different types of possibilities within investments as options and valuing them as (real) options. Situations where exercising a real option, acting on a possibility, opens new real option is not uncommon in business. These multi-stage real options are generally called compound real options. Compound real options can be found from a number of different domains, for example corporate strategy
Real options are most commonly valued with the Black-Scholes option pricing formula (Black and Scholes 1973) and the binomial option valuation method (Cox, Ross et al. 1979). These two models are based on the assumption that the processes they use to model the “markets” of the underlying asset can quite accurately create a correct probability distribution of the outcomes at maturity for the underlying asset. This is an assumption that may hold for some quite efficiently traded financial securities, but may not hold for real investments that do not have existing markets, or have markets that can by no means be said to exhibit even the weak form of market efficiency. Monte-Carlo simulation based methods allow for a wide variety of the underlying processes, see e.g. (Boyle 1977) and hence escape some of the problems of the Black-Scholes and binomial methods. There are also a number of later developed versions of the three above mentioned methods, including efforts to use fuzzy variables with these methods (Carlsson and Fullér 2003; Collan, Carlsson et al. 2003; Yoshida 2003; Muzzioli and Reynaerts 2008).

(Geske 1977; Geske and Johnson 1984) and (Geske 1979) show a method for the valuation of compound financial options using the Black-Scholes model. The Geske-models are the best known compound option valuation models,
and there are many later works that draw on them, e.g., (Lee, Yeh et al. 2008) for compound option valuation. A fuzzy compound option model based on the Geske models is presented, e.g. in (Wang and Hwang 2007).

A rather new option valuation model, designed for real option valuation is the Datar-Mathews method (DMM) that is presented in (Datar and Mathews 2004), (Mathews and Datar 2007), and (Mathews and Salmon 2007). The DMM calculates the real option value from a pay-off distribution that is derived from a probability distribution of the net present value (NPV) for a given project. It is proposed that the probability distribution of the project is generated with a (Monte-Carlo) simulation. Datar and Mathews show that the results from the DMM converge to the results from the Black-Scholes method under the Black-Scholes constraints. The DMM is able to handle many types of probability distributions, which makes it more robust than the previous methods used in option valuation. So far, to the best of our knowledge, the application of DMM in compound option valuation has not been published.

A new method for the valuation of real options called the (fuzzy) pay-off method (POM) for real option valuation was presented in (Collan 2008) and (Collan, Fullér et al. 2009). The pay-off method uses a fuzzy number for representing the expected future distribution of the project profitability outcomes, i.e., the fuzzy NPV of the project. The fuzzy NPV is the pay-off distribution for the project. The DMM shows that the probability weighted average of the outcomes of the pay-off distribution, when the negative outcomes are valued at zero is the real option value. ROV is calculated in a
similar fashion from a fuzzy NPV with the POM: ROV is the fuzzy mean of the positive side of the fuzzy NPV multiplied by the area above the positive values divided by the total area of the fuzzy NPV.

Definition 1.1. POM calculates the real option value from the fuzzy NPV as follows

\[
ROV = \frac{\int_{0}^{\infty} A(x) \, dx \times E(A_+)}{\int_{-\infty}^{\infty} A(x) \, dx}
\]

Where \( A \) stands for the fuzzy NPV, \( E(A_+) \) denotes the fuzzy mean value of the positive side of the NPV and \( \int_{-\infty}^{\infty} A(x) \, dx \) computes the area below the whole fuzzy number \( A \), and \( \int_{0}^{\infty} A(x) \, dx \) computes the area below the positive part of \( A \).

The pay-off method is different from the Black-Scholes and binomial models, because the pay-off distribution, from where the real option value is derived can be constructed directly from cash-flow scenario data for a given project. This means that the pay-off method is not dependent on a given process to model the future. This is an advantage in situations where the distribution of the future outcomes from a project, i.e., the pay-off distribution, is difficult to precisely estimate. Asymmetric distributions are also accommodated without any additional difficulty.
When compound real option valuation is considered with the pay-off method, the calculation of the compound real option value does not differ from the calculation of the value of a single real option. That is, the same method can be used for both. The difference comes from the cash-flow scenarios that underlie the compound project vs. the single project that is valued as a real option. For the compound project the cash-flow logic between, e.g., two stages may change and the change should then be visible in the cash-flows.

If process dependent real option valuation methods are used for compound projects with stages that are different in the way the cash-flows accrue (different underlying process), then there should be two different processes used to reflect the difference – this may become very complex. The pay-off method remains the same.

*Defining the fuzzy mean of the positive side of the triangular NPV distribution*

As the form of the fuzzy number may vary, the most used forms are the triangular and the trapezoidal fuzzy numbers. These are very usable forms, as they are easy to understand and can be simply defined. For the purposes of the following case illustration, we shall calculate the positive area and the possibilistic mean of the positive area of a triangular fuzzy pay-off \( A = (a, \alpha, \beta) \).

For the triangular pay-off distribution, the possibilistic mean of the positive side in the different cases (depending on where zero crosses the distribution) is calculated according to the following formulae:
1. When the whole distribution is “above zero”; when \( 0 < (a - \alpha) \)

\[
E(A_+) = \alpha + \frac{\beta - \alpha}{6}
\]

2. When the distribution is partly “above zero” so that \( a \) is above zero, but \( a-\alpha \) is below zero; when \((a - \alpha) < 0 < a\)

\[
E(A_+) = \alpha + \frac{\beta - \alpha}{6} \cdot \frac{(a - \alpha)^3}{6\alpha^2}
\]

3. When the distribution is partly “above zero” so that \( a \) is below zero, but \( a + \beta \) is above zero \((a < 0 < a + \beta)\)

\[
E(A_+) = \frac{(a + \beta)^3}{6 \cdot \beta^2}
\]

4. When the distribution is totally “below zero"

\[
E(A_+) = 0
\]

Original derivation of the possibilistic mean value is presented in (Carlsson and Fullér 2001).

In the following section we show and illustrate with an R&D related numerical case how the pay-off method can be used to value compound options. R&D is selected as the basis for the example, because in R&D projects there is often a lot of uncertainty that causes the expected cash-flow forecasting to be inaccurate. There is often also a lack of information about the drivers of the size of the future cash-flows, which tends to make forecasting even more inaccurate. Partly for the above mentioned reasons, forecasting the future with multiple cash-flow scenarios is an often used practice, when R&D valuation is made. The pay-off method is well compatible with the cash-flow scenario
approach. We continue with a discussion about the robustness and ease of use of the method for compound option valuation.

**Compound options analysis with the pay-off method: three-stage R&D case**

A company is offered the rights to a new chemical compound that holds promise for a certain coating application, but needs further exploratory research, and research in application, before a production investment and cash flows from sales are reachable. The price of the rights to the compound is 15 monetary units (mu). The question is, if the company should buy the compound at the offered price (what is the value that the company thinks the rights are worth?). The problem is a classical problem that exhibits the difficulties of decision-making based on the available incomplete information at any given moment. The simplified compound structure of the three-stage project to be valued is the following:

![Figure 1. Cumulative present value (NPV) chart for the three different scenarios](image)

Figure 1. Cumulative present value (NPV) chart for the three different scenarios

*First stage - initial investment into exploratory R&D*
The exploratory R&D duration is uncertain and we simplify it to three (min, best guess, max) scenarios 3, 5, and 7 units of time, we assume that there is a fixed cost (500 mu) per unit of time.

Second stage - investment into research in application

The application R&D duration is uncertain and we simplify it to three scenarios 1, 2, and 3 units of time, we assume that there is a fixed cost (1000 mu) per unit of time,

Third stage - production investment and operation of the investment

The production investment has a fixed cost (20000 mu) and will be made in the period directly after the completion of the application R&D and will have the duration of 1 unit of time. The sales of the product are estimated to last for 5 years. Three scenarios for operational cash-flow are estimated, for simplicity the scenarios are ±20% from the best guess scenario (best guess, optimistic, pessimistic; cash-flow tables available in appendix 1).

Figure 2. The triangular fuzzy distribution for the expected project NPV (not in scale)
For all the investments, including the R&D investments, a risk free discount rate of 5% / time unit is used, for the operational cash-flow streams we use a 15% / time unit risk adjusted discount rate. Figure 1 shows a cumulative present value chart for three scenarios that have been constructed by combining all the minimum and maximum values together for the optimistic and the pessimistic scenarios, and the best guess estimates for the best guess scenario. For a cash-flow table of the project see Appendix 1.

We simplify and expect that the three constructed scenarios represent the distribution of future outcomes for the three-stage project, so that the optimistic scenario is the maximum possible and the pessimistic scenario the minimum possible outcome of the project. We also expect that the best guess scenario is the most likely outcome from the project and assign it a full grade of membership in the set of possible NPV of the project outcomes. From the NPV scenarios we create a triangular fuzzy distribution of the project NPV, shown in figure 2. This distribution is the triangular fuzzy NPV of the project. We use the POM to calculate the real option value of the three-stage project: we get ~7486 area units (au) for the total area of the fuzzy NPV, ~935 au for the positive area of the fuzzy NPV, and ~709 mu for fuzzy mean value of the positive area of the fuzzy NPV, see figure 3. Solving for ROV we get 935 au / 7486 au * 137 mu = ~17 mu.

This indicates that (taking into consideration the simplifications) the asked price of 15 mu is lower than the real option value of the project, and the company should purchase the rights for the chemical compound in the light of
the available information and taking into consideration the value of the three-stage project as a compound real option.

*Milestone evaluation for a project with compound real options*

Let us consider that the company purchases the rights for the chemical compound and starts first stage R&D. The company has a policy to review projects at the end of each R&D stage for continuing / discontinuing them. During these evaluations the project data is usually also updated. In this simplified case the data for the project remains the same (as above).

We can see from figure 4 the situation at the end of stage 1, after the completion of the exploratory research (cash-flow tables in appendix 1). The remaining two-stage project NPV distribution yields ROV of ~5357 mu. This includes the investment cost in the second stage R&D, which means that the ROV is comparable with a cost of zero (0) and shows that the project should be continued.

![Figure 4. NPV chart for the three different scenarios at first evaluation](image-url)
Discussion and Conclusions

There are many application areas for compound real options and compound real option valuation as compound real options are to be found in many different walks of life. There are at least five different types of option valuation methods to choose from, the most often used option valuation methods: the Black-Scholes model, the binomial method, Monte-Carlo simulation based methods, the Datar-Mathews method, and the pay-off method. We have shortly presented the pay-off method and the calculation of the fuzzy mean value for the positive area of a triangular fuzzy NPV that allow us to value real options and compound real options.

We have shown with a case application that the analysis of compound real options with the pay-off method is simple and straightforward. Differences between project stages are accommodated by the expected cash-flows for the different stages, while the method used for compound real options and for single real options remains the same. Analyses with the POM are easy to execute with the most used spreadsheet software.

The scenarios presented in the case example can be made in much more detail, e.g., many of the issues simplified can be modelled in a more detailed fashion to reflect the available information about the future more closely.

The POM method can easily accommodate:
1) Different underlying processes in different project stages
2) Different timing scenarios for different project stages
3) The integration of independent project stage scenarios into project scenarios

The decision rules used with the POM are clear and easy to understand. In the case illustration an important simplification is made, when the project NPV distribution is considered to be triangular. This is not a pre-requisite for the pay-off method, as the distribution can be of any form that satisfies the requirements for a fuzzy number. It, however, is realistic to expect that the shape be easily definable, e.g., triangular or trapezoidal, for simple calculation.

Milestone evaluation is very a usual way of considering the continuation / discontinuation of staged investment projects. It has also been researched in connection with real options, e.g., see (Li 2008). Milestones permit the investors to re-evaluate the investment based on the information available at the time of evaluation. In reality information is likely to change during the life of a project. This means that the estimated cash-flows for each scenario should be adjusted. The changes in the cash-flow estimations cause the NPV distribution of the project to change, and hence affects the ROV, changes in the estimated cash-flows are easy to implement as the compound option model can be constructed with commonly used spreadsheet software.
We feel that the pay-off method allows for a user friendly application of compound real option valuation and analysis, and narrows the gap between the academia and the practitioners.

Mikael Collan, D.Sc. is a research fellow at the Institute for Advanced Management Systems Research and a project manager in the industry financed Redevelop project focusing on analysis and valuation of intellectual property rights. His research is concentrated on asset valuation and real options.

Robert Fullér is a professor at the Eötvös Lorand University in Budapest, Hungary and a Finland Distinguished Professor (FiDiPro) at the Institute for Advanced Management Systems in Turku Finland. He is a well known researcher in the area of fuzzy mathematics and applications of fuzzy logic.

M.Sc. József Mezei is working on his dissertation at the Turku School of Computer Science (TUCS) on fuzzy mathematics and the application of fuzzy logic.

References


Appendix 1: The cash-flow table for the case illustration
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**Note:** The table represents cash flows (CF), present values (PV), net present values (NPV), and cumulative present values (Cum. PV) for different scenarios (opt, bestg, pass) over time.