Hilbert Spaces Contained in Quotients of KreĐă Spaces, with Applications to Passive State/Signal Realization Theory.

Damir Z. Arov
Division of Mathematical Analysis
Institute of Physics and Mathematics
South-Ukrainian Pedagogical University
65020 Odessa, Ukraine

Olof J. Staffans
Åbo Akademi University
Department of Mathematics
FIN-20500 Åbo, Finland
http://www.abo.fi/~staffans/

Abstract

Let $Z$ be a maximal nonnegative subspace of a KreĐă space $\mathcal{K}$ with (indefinite) inner product $\langle \cdot, \cdot \rangle_{\mathcal{K}}$, let $Z_{\perp}$ be the orthogonal companion to $Z$ in $\mathcal{K}$, and let $Z_0 = Z \cap Z_{\perp}$ be the maximal neutral subspace of $Z$. Then $\langle \cdot, \cdot \rangle_{\mathcal{K}}$ induces a positive inner product in the quotient space $Z/ Z_0$, and $-\langle \cdot, \cdot \rangle_{\mathcal{K}}$ induces a positive inner product in the quotient space $Z_{\perp}/ Z_0$. These two inner product spaces way are not, in general, complete. We show that the completions of $Z/ Z_0$ and $Z_{\perp}/ Z_0$ can be identified in a natural way with certain subspaces of the quotient spaces $\mathcal{K}/ Z_{\perp}$ and $\mathcal{K}/ Z$, respectively. The construction of these subspaces is similar to the deBrange–Rovnyak construction used to realize an operator-valued Schur function in the unit disk $D$ as the characteristic function of a discrete time input/state/output system. More precisely, the completion of $Z_{\perp}/ Z_0$ can be identified with the following subspace $\mathcal{X}[Z]$ of $\mathcal{K}/ Z$.

For each $k \in \mathcal{K}$ we denote the equivalence class in $\mathcal{K}/ Z$ to which $k$ belongs by $[k] = k + Z$. Then

$\mathcal{X}[Z] = \{ [k] \in \mathcal{K}/ Z \mid \| [k] \|_{\mathcal{X}[Z]} < \infty \}$,

where the norm $\| \cdot \|_{\mathcal{X}[Z]}$ in $\mathcal{X}[Z]$ is given by

$\| [k] \|_{\mathcal{X}[Z]} = \sqrt{\sup_{z \in Z} (-\langle k - z, k - z \rangle_{\mathcal{K}})}$.

The subspace $\mathcal{X}[Z_{\perp}]$ of $\mathcal{K}/ Z_{\perp}$ is defined in an analogous fashion.

We apply the technique described above to construct three canonical passive state/signal realizations of a given passive behavior $\mathfrak{M}$, namely a) a controllable forward conservative, b) an observable backward conservative, and c) a simple conservative state/signal realization. All of these are determined unique by $\mathfrak{M}$ up to unitary similarity. The passive behavior $\mathfrak{M}$ is roughly the time-domain counterpart of a shift-invariant maximal nonnegative subspace $Z$ of the KreĐ space $\mathcal{K} := H^2(D; W)$, where $W$ is a KreĐ space. By decomposing $W$ in different ways into the direct sum of an input space and an output space and interpreting $Z$ as the graph of a shift-invariant operator we get the standard input/state/output realizations of Schur functions, Charathéodory functions, and Potapov functions in the unit disk.

References

