Feedback Representations of Critical Controls for Well-Posed Linear Systems

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This is the first part in a three part study on the suboptimal full information $H^\infty$ problem for a well-posed linear system with input space $U$, state space $H$, and output space $Y$. We define a cost function $Q(x_0, u) = \int_{\mathbb{R}^+} \langle y(s), Jy(s) \rangle_Y ds$, where $y \in L^2_{\text{loc}}(\mathbb{R}^+; Y)$ is the output of the system with initial state $x_0 \in H$ and control $u \in L^2_{\text{loc}}(\mathbb{R}^+; U)$, and $J$ is a self-adjoint operator on $Y$. The cost function $Q$ is quadratic in $x_0$ and $u$, and we suppose (in the stable case) that the second derivative of $Q(x_0, u)$ with respect to $u$ is nonsingular. This implies that, for each $x_0 \in H$, there is unique critical control $u^{\text{crit}}$ such that the derivative of $Q(x_0, u)$ with respect to $u$ vanishes at $u = u^{\text{crit}}$. We show that $u^{\text{crit}}$ can be written in feedback form whenever the input/output map of the system has a coprime factorization with a $(J, S)$-inner numerator; here $S$ is a particular self-adjoint operator on $U$. A number of properties of this feedback representation are established, such as the equivalence of the $(J, S)$-losslessness of the factorization and the positivity of the Riccati operator on the reachable subspace.