On the Discrete and Continuous Time
Infinite-Dimensional
Algebraic Riccati Equations

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The standard state space solution of the finite-dimensional continuous time quadratic cost minimization problem has a straightforward extension to infinite-dimensional problems with bounded or moderately unbounded control and observation operators. However, if these operators are allowed to be sufficiently unbounded, then a strange change takes place in one of the coefficients of the algebraic Riccati equation, and the continuous time Riccati equation begins to resemble the discrete time Riccati equation. To explain why this phenomenon must occur we discuss a particular hyperbolic PDE in one space dimension with boundary control and observation (a transmission line) that can be formulated both as a discrete time system and as a continuous time system, and show that in this example the continuous time Riccati equation can be recovered from the discrete time Riccati equation. A particular feature of this example is that the Riccati operator does not map the domain of the generator into the domain of the adjoint generator, as it does in the standard case.