On the Distributed Stable Full Information $H^\infty$ Minimax Problem

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We study the distributed parameter suboptimal full information $H^\infty$ problem for a stable well-posed linear system with control $u$, disturbance $w$, state $x$, and output $y$. Here $u$, $w$, and $y$ are $L^2$-signals on $(0,\infty)$ with values in the Hilbert spaces $U$, $W$, and $Y$, and the state $x$ is a continuous function of time with values in the Hilbert space $H$. The problem is to determine if there exists a (dynamic) $\gamma$-suboptimal feedforward compensator, i.e., a compensator $U$ such that the choice $u = Uw$ makes the norm of the input/output map from $w$ to $y$ less than a given constant $\gamma$. A sufficient condition for the existence of a $\gamma$-suboptimal compensator is that an appropriately extended input/output map of the system has a $(J,S)$-inner-outer factorization of a special type, and if the control and disturbance spaces are finite-dimensional and the system has an $L^1$ impulse response, then this condition is also necessary. Moreover, in this case there exists a central state feedback/feedforward controller, which can be used to give a simple parameterization of the set of all $\gamma$-suboptimal compensators. Our proofs use a game theory approach.