Passive and conservative state/signal systems in continuous time

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In this talk we discuss passive and conservative state/signal systems in continuous time. Such a system can be used to model, e.g., a passive linear electrical circuit containing lumped and/or distributed resistances, capacitors, inductors, and wave guides, etc. Most of the standard partial differential equations appearing in physics on can be written in state/signal form. A passive state/signal system consists of three components: A) an internal Hilbert state space $X$, B) a Krein signal space $W$ through which the system interacts with the external world, and C) a generating subspace $V$ of the product space $X \times X \times W$. The generating subspace is required to be maximally nonnegative with respect to a certain “energy” inner product and to satisfy an extra nondegeneracy condition. We denote this system by $\Sigma = (V; X, W)$. The set of all classical trajectories of $\Sigma$ on some interval $I$ consists of a continuously differentiable $X$-valued state component $x$ and a continuous $W$-valued signal component $w$ satisfying

$$(\dot{x}(t), x(t), w(t)) \in V, \quad t \in I.$$ 

The set of all generalized trajectories of $\Sigma$ is obtained from the family of all classical trajectories by a standard approximation procedure.

By the future behavior of $\Sigma$ we mean the set of all signal parts $w$ of all stable trajectories $(x, w)$ of $\Sigma$ on $[0, \infty)$ satisfying the extra condition $x(0) = 0$. This set is a right-shift invariant subspace of $L^2([0, \infty); W)$ and it is maximal nonnegative with respect to the Krein space inner product in $L^2([0, \infty); W)$ inherited from $W$. Such a subspace is called a passive future behavior. Each passive future behavior can be realized as the future behavior of a passive state/signal system $\Sigma$, and it is possible to require $\Sigma$ to have, for example, one of the following three sets of properties: a) $\Sigma$ is observable and co-energy preserving; b) $\Sigma$ is controllable and energy
preserving; c) $\Sigma$ is simple and conservative. Realizations within one of these classes are uniquely determined by the given future behavior. Furthermore, it is possible to construct canonical realizations, i.e., realizations that satisfy a), b), or c), and which are uniquely determined by the given data.

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